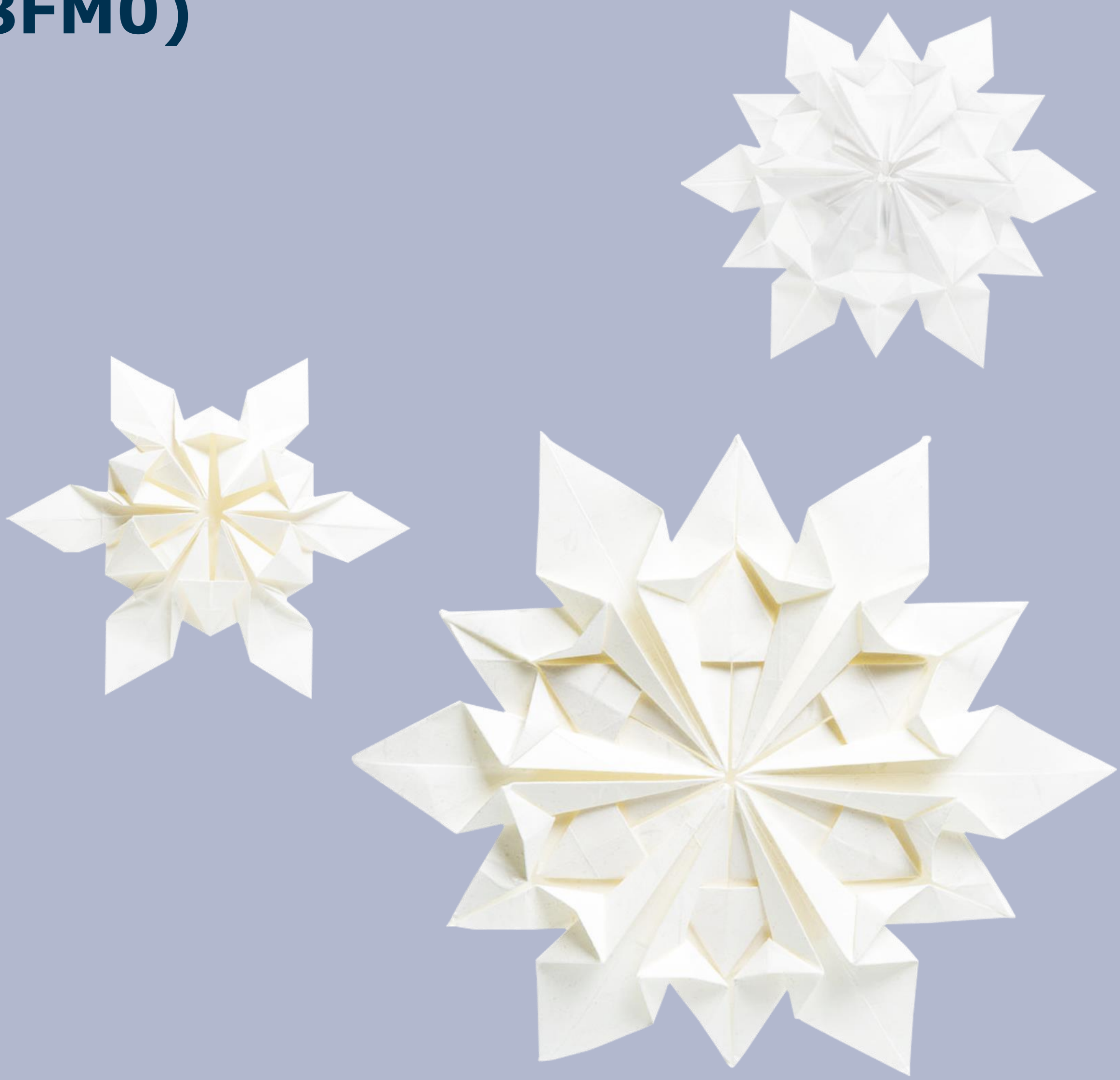
Pearson Edexcel Level 3 Advanced Subsidiary GCE in Further Mathematics (8FM0)



Sample Assessment Materials Model Answers – Further Pure Mathematics 1&2

First teaching from September 2017
First certification from June 2018

Sample Assessment Materials Model Answers – Further Pure Mathematics 1&2

Contents

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Introduction

This booklet has been produced to support mathematics teachers delivering the new Pearson Edexcel Level 3 Advanced Subsidiary GCE in Mathematics (8FMO) specification for first teaching from September 2017.

This booklet looks at Sample Assessment Materials for AS Further Mathematics qualification, specifically at further pure mathematics 1 and 2 questions, and is intended to offer model solutions with different methods explored.

Content of Further Pure Mathematics 1&2

Content	**AS level content**
	Further Pure Mathematics 1
Further trigonometry	The *t*-formulae. Applications of t-formulae to trigonometric identities. Applications of *t*-formulae to solve trigonometric equations.
Coordinate systems	Cartesian equations for the parabola and rectangular hyperbola. Parametric equations for the parabola and rectangular hyperbola. The focus-directrix property of the parabola. Tangents and normals to these curves. Simple loci problems.
Further vectors	The vector product **a**× **b** of two vectors. Applications of the vector product. The scalar triple product **a**.**b**×**c**
Numerical Methods	Numerical solution of first order and second order differential equations.
Inequalities	The manipulation and solution of algebraic inequalities and inequations.
	Further Pure Mathematics 2
Groups	The Axioms of a group. Examples of groups. Cayley tables. Cyclic groups. The order of a group and the order of an element. Subgroups. Lagrange's theorem.
Further matrix algebra	Eigenvalues and eigenvectors of 2 × 2 matrices. Reduction of matrices to diagonal form. The use of the Cayley-Hamilton theorem.
Further complex numbers	Further loci and regions in the Argand diagram.
Number theory	An understanding of the division theorem and its application to the Euclidean Algorithm and congruences. Bezout's identity. Modular arithmetic. Understanding what is meant by two integers *a* and *b* to be congruent modulo an integer *n*. Properties of congruences. Divisibility Tests.
Further sequences and series	First order recurrence relations. The solution of recurrence relations to obtain closed forms. Proof by induction of closed forms.

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AS Further Pure Mathematics 1

Question 1

(a) Use the substitution $t = \tan \dfrac{x}{2}$ to show that

$$\sec x - \tan x = \frac{1-t}{1+t}, \qquad x \neq (2n+1)\frac{\pi}{2}, \quad n \in \mathbb{Z}.$$

(3)

$\tan x = \dfrac{2t}{1-t^2}$, $\cos x = \dfrac{1-t^2}{1+t^2}$, $\sec x = \dfrac{1+t^2}{1-t^2}$

$\sec x - \tan x = \dfrac{1+t^2}{1-t^2} - \dfrac{2t}{1-t^2}$ M1

$= \dfrac{1-2t+t^2}{1-t^2}$ M1

$= \dfrac{(1-t)^2}{(1-t)(1+t)}$

$= \dfrac{1-t}{1+t}$ A1

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(b) Use the substitution $t = \tan \frac{x}{2}$ and the answer to part (a) to prove that

$$\frac{1-\sin x}{1+\sin x} \equiv (\sec x - \tan x)^2, \qquad x \neq (2n+1)\frac{\pi}{2}, \quad n \in \mathbb{Z}.$$

(3)

$\sin x \frac{2t}{1+t^2}$

$$\frac{1-\sin x}{1+\sin x} = \frac{1-\frac{2t}{1+t^2}}{1+\frac{2t}{1+t^2}}$$ M1

$$= \frac{1+t^2-2t}{1+t^2+2t}$$ M1

$$= \frac{(1-t)^2}{(1+t)^2}$$

$$= \left(\frac{1-t}{1+t}\right)^2$$

$$= (\sec x - \tan x)^2$$ A1

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Question 2

The value, V hundred pounds, of a particular stock t hours after the opening of trading on a given day is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{V^2 - t}{t^2 + tV}, \qquad 0 < t < 8.5.$$

A trader purchases £300 of the stock one hour after the opening of trading.

Use two iterations of the approximation formula $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_0}{h}$ to estimate, to the nearest £, the value of the trader's stock half an hour after it was purchased.

(6)

In this context $y = V$ and $x = t$, the approximation is $\left(\frac{dV}{dt}\right)_0 = \frac{V_1 - V_0}{h}$

£300 purchased 1 hour after opening: $V_0 = 3,\ t_0 = 1$.
Half an hour after purchase: $t_2 = 1.5$, so step $h = 0.5/2 = 0.25$ B1

$$\left(\frac{dV}{dt}\right)_0 \approx \frac{V_0^2 - t_0}{t_0^2 + t_0 V_0}$$

$$= \frac{3^2 - 1}{1^2 + 1 \times 3} = 2$$ M1

$$V_1 \approx V_0 + h\left(\frac{dV}{dt}\right)_0$$

$= 3 + 0.25 \times 2$ M1

$= 3.5$ A1

$$\left(\frac{dV}{dt}\right)_1 \approx \frac{V_1^2 - t_1}{t_1^2 + t_1 V_1}$$

$$= \frac{3.5^2 - 1.25}{1.25^2 + 1.25 \times 3.5}$$ M1

$$= \frac{176}{95}$$

$$V_2 \approx V_1 + h\left(\frac{dV}{dt}\right)_1$$

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$= 3.5 + 0.25 \times \frac{176}{95}$

$= 3.963...$

$= £396$ A1

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Question 3

Use algebra to find the set of values of x for which

$$\frac{1}{x} < \frac{x}{x+2}.$$

(6)

$$\frac{1}{x} - \frac{x}{x+2} < 0$$

$$\frac{1(x+2) - x \times x}{x(x+2)} < 0$$ M1

$$\frac{2 + x - x^2}{x(x+2)} < 0$$

$$\frac{x^2 - x - 2}{x(x+2)} > 0$$

$$\frac{(x-2)(x+1)}{x(x+2)} > 0$$ M1

There are four critical values, –2, –1, 0, 2. (any 2) A1
(all) A1

	$x+2$	$x+1$	x	$x-2$	Overall
$x < -2$	–	–	–	–	+
$-2 < x < -1$	+	–	–	–	–
$-1 < x < 0$	+	+	–	–	+
$0 < x < 2$	+	+	+	–	–
$x > 2$	+	+	+	+	+

So require $x < -2$, $-1 < x < 0$, $x > 2$ M1

In set notation:
$\{x \in \mathbb{R} : x < -2\} \cup \{x \in \mathbb{R} : -1 < x < 0\} \cup \{x \in \mathbb{R} : x > 2\}$ A1

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Alternative: (for first 2 marks)

$$\frac{1}{x} - \frac{x}{x+2} < 0$$

Multiply through by $x^2(x + 2)^2$:

$x(x + 2)^2 - x^3(x + 2) < 0$ M1

$x(x + 2)[x + 2 - x^2] < 0$

$x(x + 2)(x^2 - x - 2) > 0$

$x(x + 2)(x - 2)(x + 1) > 0$ M1

Then as before.

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Question 4

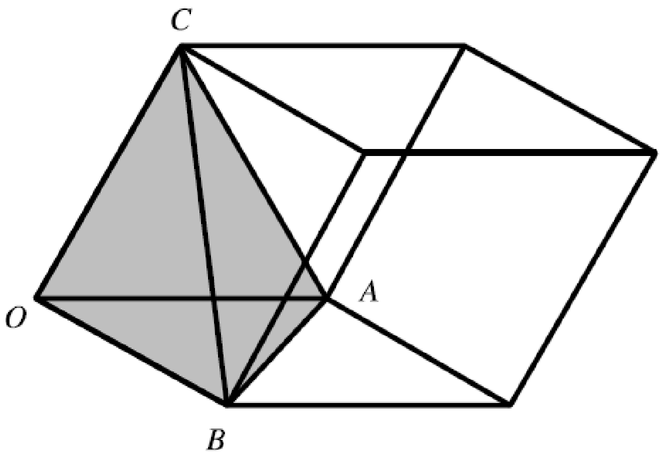

Figure 1

Figure 1 shows a sketch of a solid sculpture made of glass and concrete. The sculpture is modelled as a parallelepiped.

The sculpture is made up of a concrete solid in the shape of a tetrahedron, shown shaded in Figure 1, whose vertices are $O(0, 0, 0)$, $A(2, 0, 0)$, $B(0, 3, 1)$ and $C(1, 1, 2)$,where the units are in metres. The rest of the solid parallelepiped is made of glass which is glued to the concrete tetrahedron.

(a) Find the surface area of the glued face of the tetrahedron.

(4)

The glued face is the triangle ABC.

Area $= \frac{1}{2}|\mathbf{AB} \times \mathbf{AC}|$ M1

$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = (3\mathbf{j} + \mathbf{k}) - (2\mathbf{i}) = -2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$
$\mathbf{AC} = \mathbf{OC} - \mathbf{OA} = (\mathbf{i} + \mathbf{j} + 2\mathbf{k}) - (2\mathbf{i}) = -\mathbf{i} + \mathbf{j} + 2\mathbf{k}$

$$\frac{1}{2}|\mathbf{AB} \times \mathbf{AC}| = \frac{1}{2}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 3 & 1 \\ -1 & 1 & 2 \end{vmatrix}$$ M1

$$= \frac{1}{2}|(3 \times 2 - 1 \times 1)\mathbf{i} - (-2 \times 2 - 1 \times -1)\mathbf{j} + (-2 \times 1 - 3 \times -1)\mathbf{k}|$$

$$= \frac{1}{2}|5\mathbf{i} + 3\mathbf{j} + \mathbf{k}|$$ M1

$$= \frac{1}{2}\sqrt{5^2 + 3^2 + 1^2}$$

$$= \frac{1}{2}\sqrt{35}$$ A1

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Alternative:

The glued face is the triangle *ABC*.

Area = $\frac{1}{2}\sqrt{|\mathbf{AB}|^2|\mathbf{AC}|^2 - (\mathbf{AB.AC})^2}$ M1

$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = (3\mathbf{j} + \mathbf{k}) - (2\mathbf{i}) = -2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$
$\mathbf{AC} = \mathbf{OC} - \mathbf{OA} = (\mathbf{i} + \mathbf{j} + 2\mathbf{k}) - (2\mathbf{i}) = -\mathbf{i} + \mathbf{j} + 2\mathbf{k}$

$|\mathbf{AB}|^2 = (-2)^2 + 3^2 + 1^2 = 14$
$|\mathbf{AC}|^2 = (-1)^2 + 1^2 + 2^2 = 6$
$\mathbf{AB.AC} = (-1\times-2) + (3\times1) + (1\times2) = 7$ M1

Area = $\frac{1}{2}\sqrt{14\times6-7^2}$ M1

$= \frac{1}{2}\sqrt{35}$ A1

(b) Find the volume of glass contained in this parallelepiped. **(5)**

Total volume of parallelepiped is found from the scalar triple product: $|\mathbf{OA}.(\mathbf{OB}\times\mathbf{OC})|$

Volume of concrete tetrahedron is found from the scalar triple product: $\frac{1}{6}|\mathbf{OA}.(\mathbf{OB}\times\mathbf{OC})|$

M1

$|\mathbf{OA}.(\mathbf{OB}\times\mathbf{OC})| = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 1 & 1 & 2 \end{vmatrix}$ M1

$= 2(3\times2 - 1\times1) - 0 + 0$
$= 10$

Volume of concrete tetrahedron = $\frac{1}{6}\times10 = \frac{5}{3}$ A1

Volume of glass = volume of parallelepiped – volume of tetrahedron

$= 10 - \frac{5}{3}$ M1

$= \frac{25}{3}$ A1

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Alternative:

Volume of concrete tetrahedron = $\frac{1}{3}$ (base area) × (height)

To find height, find a vector perpendicular to both **OA** and **OB**.
$(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}).(2\mathbf{i}) = 0 \qquad x = 0$
$(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}).(3\mathbf{j} + \mathbf{k}) = 0 \qquad 3y + z = 0$
so e.g. $-\mathbf{j} + 3\mathbf{k}$ is perpendicular to both. M1

OA.OB = 0 so these vectors are also perpendicular

Area of base $AOB = \frac{1}{2}|\mathbf{OA}|\times|\mathbf{OB}|$

$= \frac{1}{2} \times 2 \times \sqrt{3^2 + 1^2} = \sqrt{10}$ A1

Let M = point vertically below C, so height of tetrahedron = $|\mathbf{MC}|$
$\mathbf{OC} - \mathbf{MC} = \mathbf{OM}$
$\mathbf{OM} = \mu\mathbf{OA} + \lambda\mathbf{OB}$
$(\mathbf{i} + \mathbf{j} + 2\mathbf{k}) - p(-\mathbf{j} + 3\mathbf{k}) = \mu(2\mathbf{i}) + \lambda(3\mathbf{j} + \mathbf{k})$
$1 = 2\mu$
$1 + p = 3\lambda$
$2 - 3p = \lambda$
solve to give $p = \frac{1}{2}$

height $= \frac{1}{2}|-\mathbf{j} + 3\mathbf{k}| = \frac{1}{2}\sqrt{(-1)^2 + 3^2} = \frac{1}{2}\sqrt{10}$ M1

Volume of concrete tetrahedron $= \frac{1}{3}\sqrt{10} \times \frac{1}{2}\sqrt{10} = \frac{5}{3}$

Total volume of parallelepiped = 6 × volume of tetrahedron
Volume of glass = 5 × volume of tetrahedron

$= 5 \times \frac{5}{3}$ M1

$= \frac{25}{3}$ A1

(c) Give a reason why the volume of concrete predicted by this model may not be an accurate value for the volume of concrete that was used to make the sculpture.	**(1)**

e.g.
The glued surfaces may distort the shapes or reduce the amount of concrete.
Measurements in metres may not be accurate.
The surface of the concrete tetrahedron may not be smooth.
Pockets of air may form when the concrete is being poured. B1

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Question 5

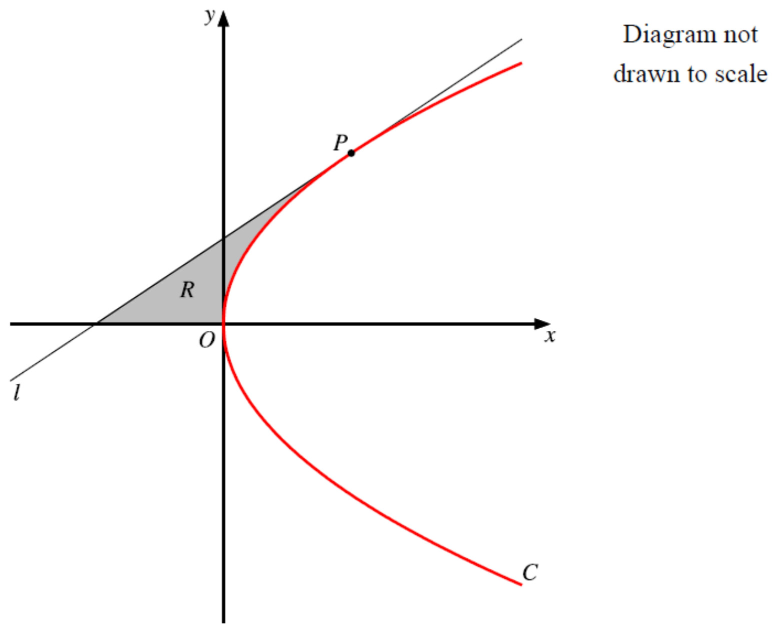

Figure 2

[You may quote without proof that for the general parabola $y^2 = 4ax$, $\dfrac{dy}{dx} = \dfrac{2a}{y}$.]

The parabola C has equation $y^2 = 16x$.

(a) Deduce that the point $P(4p^2, 8p)$ is a general point on C.

(1)

$y^2 = (8p)^2 = 64p^2$

$16x = 16(4p^2) = 64p^2$

so $P(4p^2, 8p)$ is a general point on C. B1

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The line l is the tangent to C at the point P.

(b) Show that an equation for l is $py = x + 4p^2$.

(3)

$y^2 = 4 \times 4x$ so $a = 4$ M1

$$\frac{dy}{dx} = \frac{2 \times 4}{y} = \frac{8}{y}$$

l: $y - y_1 = m(x - x_1)$

$y - 8p = \left(\frac{8}{8p}\right)(x - 4p^2)$ M1

$py - 8p^2 = x - 4p^2$

$py = x + 4p^2$ A1

The finite region R, shown shaded in Figure 2, is bounded by the line l, the x-axis and the parabola C.

The line l intersects the directrix of C at the point B, where the y coordinate of B is $\frac{10}{3}$.

Given that $p > 0$,

(c) show that the area of R is 36.

(8)

Main Method:

directrix: $x = -a$ so $x = -4$

$B(-4, \frac{10}{3})$

Put into line l:

$\frac{10}{3}p = -4 + 4p^2$ M1

$12p^2 - 10p - 12 = 0$

$6p^2 - 5p - 6 = 0$

$(2p - 3)(3p + 2)$

$p = \frac{3}{2}$ (not $-\frac{2}{3}$ as $p > 0$) M1

l cuts x axis when $y = 0$, so $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2$

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$x = -4\left(\frac{9}{4}\right)$ M1

$x = -9$ A1

$p = \frac{3}{2}$ gives $P(9, 12)$

Area (R) = Area of triangle – Area under curve

$= \frac{1}{2}(9-(-9))(12) - \int_4^9 x^{\frac{1}{2}}\,dx$ M1

$= 108 - \left[\frac{8}{3}x^{\frac{3}{2}}\right]_0^9$ M1A1

$= 108 - \frac{8}{3}\left(9^{\frac{3}{2}} - 0\right)$

$= 108 - 72$

$= 36$ A1

<u>Alternative 1:</u>

First 2 marks as main method. M1
M1

Put $p = \frac{3}{2}$ into l:

$\frac{3}{2}y = x + 4\left(\frac{3}{2}\right)^2$ M1

$x = \frac{3}{2}y - 9$ A1

$p = \frac{3}{2}$ gives $P(9, 12)$

Area (R) = Integrate with respect to y, area between curve & line

$= \int_0^{12} \frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)dy$ M1

$= \left[\frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y\right]_0^{12}$ M1

A1

$= \left(\frac{1}{48}(12)^3 - \frac{3}{4}(12)^2 + 9(12)\right) - 0$

$= 36 - 108 + 108$

$= 36$ A1

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Alternative 2:

First 4 marks as main method. M1, M1, M1, A1

$p = \frac{3}{2}$ gives $P(9, 12)$

l is $\frac{3}{2}\ (y) = x + \left(\frac{3}{2}\right)^2$

$y = \frac{2}{3}x + 6$

l cuts y axis when $x = 0$, so $y = 6$

Area (R) = Area of triangle (quadrant 2) + area between line & curve (quadrant 1)

$$= \frac{1}{2}(9)(6) + \int_0^9 \left(\frac{2}{3}x + 6\right) - 4x^{\frac{1}{2}}\,dx$$ M1

$$= 27 + \left[\frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}\right]_0^9$$ M1A1

$$= 27 + \left(\frac{1}{3}(9)^2 + 6(9) - \frac{8}{3}(9)^{\frac{3}{2}}\right) - 0$$

$= 27 + (27 + 54 – 72)$

$= 27 + 9$

$= 36$ A1

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AS Further Pure Mathematics 2

Question 6

Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 6 & 4 \end{pmatrix},$$

(a) find the characteristic equation of the matrix **A**.

(2)

The characteristic equation of matrix **A** is found from
det $(\mathbf{A} - \lambda\mathbf{I}) = 0$

$$\det\begin{pmatrix} 3-\lambda & 1 \\ 6 & 4-\lambda \end{pmatrix} = 0$$

$(3-\lambda)(4-\lambda) - 1\times 6 = 0$ M1

$12 - 7\lambda + \lambda^2 - 6 = 0$

$\lambda^2 - 7\lambda + 6 = 0$ A1

(b) Hence show that $\mathbf{A}^3 = 43\mathbf{A} - 42\mathbf{I}$.

(3)

Using the Cayley Hamilton Theorem

$\mathbf{A}^2 - 7\mathbf{A} + 6\mathbf{I} = 0$ B1

$\mathbf{A}^2 = 7\mathbf{A} - 6\mathbf{I}$

$\mathbf{A}^3 = 7\mathbf{A}^2 - 6\mathbf{A}$ M1

$\mathbf{A}^3 = 7(7\mathbf{A} - 6\mathbf{I}) - 6\mathbf{A}$

$\mathbf{A}^3 = 49\mathbf{A} - 42\mathbf{I} - 6\mathbf{A}$

$\mathbf{A}^3 = 43\mathbf{A} - 42\mathbf{I}$ A1

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Question 7

(i) Without performing any division, explain why 8184 is divisible by 6.

(2)

Add digits: $8+1+8+4 = 21$
Because 21 is divisible by 3, 8184 is also divisible by 3. M1

8184 is even, so is divisible by 2.
Since 8184 is divisible by both 3 and 2, it is divisible by 6. A1

(ii) Use the Euclidean algorithm to find integers a and b such that $27a + 31b = 1$.

(4)

Euclidean Algorithm:

$31 = 27 \times 1 + 4$

$27 = 4 \times 6 + 3$ M1

$4 = 3 \times 1 + 1$ A1

$3 = 1 \times 3$
so highest common factor of 31 & 27 is 1

$1 = 4 - 3 \times 1$
$1 = 4 - (27 - 4 \times 6) \times 1$
$1 = 4 - 27 + 24$
$1 = 28 - 27$
$1 = 4 \times 7 - 27 \times 1$ M1

$1 = (31 - 27 \times 1) \times 7 - 27 \times 1$
$1 = 31 \times 7 - 27 \times 7 - 27 \times 1$
$1 = 31 \times 7 - 27 \times (7 + 1)$
$1 = 31 \times 7 - 27 \times 8$
$1 = 31a - 27b$

$a = 7$, $b = -8$ A1

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Question 8

A curve C is described by the equation

$$|z - 9 + 12\text{i}| = 2|\,z|.$$

(a) Show that C is a circle, and find its centre and radius.

(4)

Let $z = x + y\text{i}$

$|x + y\text{i} - 9 + 12\text{i}| = 2|x + y\text{i}|$

$|(x - 9) + (y + 12)\text{i}| = 2|x + y\text{i}|$

$|(x - 9) + (y + 12)\text{i}|^2 = 4|x + y\text{i}|^2$

$(x - 9)^2 + (y + 12)^2 = 4(x^2 + y^2)$ M1

$x^2 - 18x + 81 + y^2 + 24y + 144 = 4x^2 + 4y^2$

$3x^2 + 3y^2 + 18x - 24y - 225 = 0$

This is the equation of a circle. A1

$x^2 + y^2 + 6x - 8y - 75 = 0$

$(x + 3)^2 - 9 + (y - 4)^2 - 16 = 75$

$(x + 3)^2 + (y - 4)^2 = 10^2$ M1

Centre is $(-3, 4)$, radius $= 10$ A1

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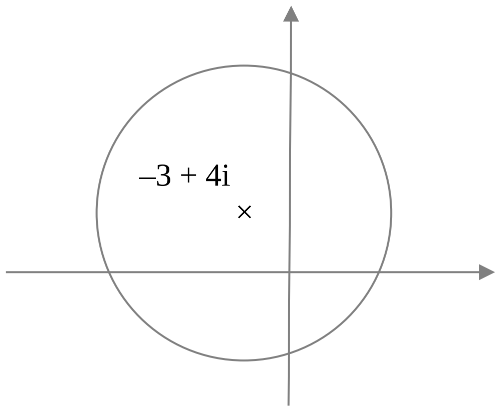
(b) Sketch C on an Argand diagram.

(2)



M1
A1

Given that w lies on C,

(c) find the largest value of a and the smallest value of b that must satisfy $a \leq \text{Re}(w) \leq b$.

(2)

Values range from $-3 - 10$ to $-3 + 10$ M1

i.e. from -13 to 7

So $-13 \leq \text{Re}(w) \leq 7$ A1

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Question 9

The operation * is defined on the set $S = \{0, 2, 3, 4, 5, 6\}$ by $x*y = x + y - xy \pmod 7$.

*	0	2	3	4	5	6
0						
2		0				
3						5
4						
5		4				
6						

(a) (i) Copy and complete the Cayley table shown above

(ii) Show that S is a group under the operation *

(You may assume the associative law is satisfied.)

(6)

(i)
If $x = 0$, $0*y = 0 + y - 0y = y$
so $0*2 = 2$, $0*3 = 3$ etc

If $y = 0$, $x*0 = x + 0 - x0 = x$
so $2*0 = 2$, $3*0 = 3$ etc

As $3*6 = 5$ then $6*3 = 5$ as this operation is commutative.
similarly $5*2 = 2*5 = 4$ M1

$2*3 = 2 + 3 - 2 \times 3 = -1 = 6 \pmod 7$
$2*4 = 2 + 4 - 2 \times 4 = -2 = 5 \pmod 7$
$2*6 = 2 + 6 - 2 \times 6 = -4 = 3 \pmod 7$
$3*3 = 3 + 3 - 3 \times 3 = -3 = 4 \pmod 7$
$3*4 = 3 + 4 - 3 \times 4 = -5 = 2 \pmod 7$
$3*5 = 3 + 5 - 3 \times 5 = -7 = 0 \pmod 7$ M1
$4*4 = 4 + 4 - 4 \times 4 = -8 = 6 \pmod 7$
$4*5 = 4 + 5 - 4 \times 5 = -11 = 3 \pmod 7$
$4*6 = 4 + 6 - 4 \times 6 = -14 = 0 \pmod 7$
$5*5 = 5 + 5 - 5 \times 5 = -15 = 6 \pmod 7$
$5*6 = 5 + 6 - 5 \times 6 = -19 = 2 \pmod 7$
$6*6 = 6 + 6 - 6 \times 6 = -24 = 4 \pmod 7$

$3*2 = 2*3$ etc
The completed table is

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*	0	2	3	4	5	6
0	0	2	3	4	5	6
2	2	0	6	5	4	3
3	3	6	4	2	0	5
4	4	5	2	6	3	0
5	5	4	0	3	6	2
6	6	3	5	0	2	4

A1

(ii)
The identity element is 0.

There is closure, the results of all calculations are also elements in the group. M1

All elements have inverses: 3 & 5 are inverses, 4 & 6 are inverses, 2 is self-inverse, 0 is identity so self-inverse. M1

The associative law is assumed.

All four axioms are satisfied so set *S* forms a group. A1

(b) Show that the element 4 has order 3.

(2)

4*4*4 = (4*4)*4
= 6*4 M1
= 0

0 is the identity so 4 has order 3. A1

(c) Find an element which generates the group and express each of the elements in terms of this generator.

(3)

Either:	**Or:**	
3 has order 6 so generates the group	5 has order 6 so generates the group	M1
$3^1 = 3$	$5^1 = 5$	
$3^2 = 3*3 = 4$	$5^2 = 5*5 = 6$	
$3^3 = 3*3*3 = 4*3 = 2$	$5^3 = 5*5*5 = 6*5 = 2$	
$3^4 = 3*3*3*3 = 2*3 = 6$	$5^4 = 5*5*5*5 = 2*5 = 4$	A1
$3^5 = 6*3 = 5$	$5^5 = 4*5 = 3$	
$3^6 = 5*3 = 0$	$5^6 = 3*5 = 0$	A1

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Question 10

A population of deer on a large estate is assumed to increase by 10% during each year due to natural causes.

The population is controlled by removing a constant number, Q, of the deer from the estate at the end of each year.

At the start of the first year there are 5000 deer on the estate.

Let P_n be the population of deer at the end of year n.

(a) Explain, in the context of the problem, the reason that the deer population is modelled by the recurrence relation

$$P_n = 1.1\,P_{n-1} - Q, \qquad P_0 = 5000, \qquad n \in \mathbb{Z}^+.$$

(3)

P_{n-1} is the population at the end of year n–1 and this is increased by 10% by the end of year n, so is multiplied by 110% = 1.1 to give $1.1 \times P_{n-1}$ as new population by natural causes.
B1

Q is subtracted from $1.1 \times P_{n-1}$ as Q is the number of deer removed from the estate.
B1

So $P_n = 1.1 \times P_{n-1} - Q$,
$P_0 = 5000$ as population at the start is 5000 and n is a positive integer. B1

(b) Prove by induction that $P_n = (1.1)^n(5000 - 10Q) + 10Q, \quad n \geq 0$.

(5)

Let $n = 0$,
$P_0 = (1.1)^0(5000 - 10Q) + 10Q = 5000$
so result is true for $n = 0$. B1

Assume true for $n = k$.
$P_k = (1.1)^k(5000 - 10Q) + 10Q$

then
$P_{k+1} = 1.1 \times [(1.1)^k(5000 - 10Q) + 10Q] - Q$ M1
A1

$= (1.1)^{k+1}(5000 - 10Q) + 11Q - Q$
$= (1.1)^{k+1}(5000 - 10Q) + 10Q$ A1

This implies the result holds for $n = k + 1$ and so by induction
$P_n = (1.1)^n(5000 - 10Q) + 10Q$ is true for all integers $n \geq 0$. B1

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(c) Explain how the long term behaviour of this population varies for different values of Q.

(2)

Let $P_{n-1} = P_n = 5000$
$5000 = 1.1 \times 5000 - Q$
$Q = 5500 - 5000 = 500$

For $Q < 500$ the population of deer will grow.
For $Q > 500$ the population of deer will fall. B1
For $Q = 500$ the population of deer remains steady at 5000. B1

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